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APPLICATION OF SOFT P-OPEN SET TO BINARY SOFT STRUCTURES

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ABSTRACT

The main aim of this paper is to introduce a single structure which carries the subsets of X as well as the subsets of Y under the parameter E for studying the information about the ordered pair of soft subsets of X and Y . Such a structure is called a binary soft structure from X to Y . The purpose of this paper is to introduce certain binary soft weak axioms that are analogous to the axioms of topology.

KEYWORDS

Binary soft topology, binary soft weak open sets, binary soft weak closed sets, binary soft weak separation axioms and binary soft T_0 space with respect to coordinates.

1. INTRODUCTION

The concept of soft sets was first introduced by Molodtsov in 1999 as a general mathematical technique for dealing with uncertain substances [1]. In, Molodtsov magnificently applied the soft theory in numerous ways, such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration, probability, theory of measurement, and so on [1,2]. Point soft set topology deals with a non-empty set X together with a collection τ of sub set X under some set of parameters satisfying certain conditions. Such a collection τ is called a soft topological structure on X .

In 2016 Ahu Acikgöz and Nihal Tas introduced the notion of binary soft set Theory on two master sets and studied some basic characteristics [3]. In prolongation, a group researcher planned the idea of binary soft topology and linked fundamental properties which are defined over two master sets with appropriate parameters [4]. Other researchers threw their detailed discussion on Binary Soft Topological [5]. Dr. A. Kalachelvi and P.H. Malini beautifully discussed Application of Fuzzy Soft Sets to Investment Decision and also discussed some more results related to this particular field [6]. N. Y. Özgür and N. Taş, studied some more Application of Fuzzy Soft Sets to Investment Decision Making Problem [7]. N. Taş, N. Y. Özgür and P. Demir worked over An Application of Soft Set and Fuzzy Soft Set Theories to Stock Management J. C. R. Alcantud et al carefully discussed Valuation Fuzzy Soft Sets: A Flexible Fuzzy Soft Set Based Decision-Making Procedure for the Valuation of Assets N. Çağman, S [8-10].

Enginoğlu attractively explored Soft Matrix Theory and some very basic results related to it and Its Decision Making In continuation, in the present paper binary soft topological structures known as soft weak structures with respect to first coordinate as well as with respect to second coordinate are defined. Moreover, some basic results related to these structures are also planted in this paper. The same structures are defined over soft points of binary soft topological structure and related results are also reflected here with respect to ordinary and soft points.

2. PRELIMINARIES

Definition 1: [11]. Let X be an initial universe and let E be a set of parameters. Let $P(X)$ denote the power set of X and let A be a non-empty subset of E . A pair (F, A) is called a soft set over X , where F is a mapping given by: $A \rightarrow P(X)$. In other words, a soft set over X is a parameterized family of subsets of the universe X . For $e \in A$, $F(e)$ may be considered as the set of e -approximate elements of the soft set (F, A) . Clearly, a soft set

is not a set.

Let U_1, U_2 be two initial universe sets and E be a set of parameters.

Let $P(U_1), P(U_2)$ denote the power set of U_1, U_2 respectively. Also, let $A, B, C \subseteq E$.

Definition 2: [3]. A pair (F, A) is said to be a binary soft set over U_1, U_2 where F is defined as below:

$F: A \rightarrow P(U_1) \times P(U_2)$, $F(e) = (X, Y)$ for each $e \in A$ such that $X \subseteq U_1, Y \subseteq U_2$

Definition 3: [3]. A binary soft set (F, A) over U_1, U_2 is called a binary absolute soft set, denoted by \tilde{A} if $F(e) = (U_1, U_2)$ for each $e \in A$.

Definition 4: [3]. The intersection of two binary soft sets of (F, A) and (G, B) over the common U_1, U_2 is the binary soft set (H, C) , where $C = A \cap B$ and for all $e \in C$

$$H(e) = \begin{cases} (X_1, Y_1) & \text{if } e \in A - B \\ (X_2, Y_2) & \text{if } e \in B - A \\ (X_1 \cup X_2, Y_1 \cup Y_2) & \text{if } e \in A \cap B \end{cases}$$

Such that $F(e) = (X_1, Y_1)$ for each $e \in A$ and $G(e) = (X_2, Y_2)$ for each $e \in B$. We denote it $(F, A) \tilde{\cap} (G, B) = (H, C)$

Definition 5: [3]. The intersection of two binary soft sets (F, A) and (G, B) over a common U_1, U_2 is the binary soft set (H, C) , where $C = A \cap B$, and $H(e) = (X_1 \cap X_2, Y_1 \cap Y_2)$ for each $e \in C$ such that $F(e) = (X_1, Y_1)$ for each $e \in A$ and $G(e) = (X_2, Y_2)$ for each $e \in B$. We denote it as $(F, A) \tilde{\cap} (G, B) = (H, C)$

Definition 6: [3]. Let (F, A) and (G, B) be two binary soft sets over a common U_1, U_2 . (F, A) is called a binary soft subset of (G, B) if
(i) $A \subseteq B$,
(ii) $X_1 \subseteq X_2$ and $Y_1 \subseteq Y_2$ Such that $F(e) = (X_1, Y_1)$, $G(e) = (X_2, Y_2)$ for each $e \in A$. We denote it as $(F, A) \tilde{\subseteq} (G, B)$.

Definition 7: [3]. A binary soft set (F, A) over U_1, U_2 is called a binary null soft set, denoted by (\emptyset, A) if $F(e) = (\emptyset, \emptyset)$ for each $e \in A$.

Definition 8: [3]. The difference of two binary soft sets (F, A) and (G, B)

over the Common U_1, U_2 is the binary soft set (H, A) , where $H(e) = (X_1 - X_2, Y_1 - Y_2)$ for each $e \in A$ such that $(F, A) = (X_1, Y_1)$ and $(G, A) = (X_2, Y_2)$.

Definition 9: [4]. Let τ_Δ be the collection of binary soft sets over U_1, U_2 then τ_Δ is said to be a binary soft topology on U_1, U_2 if

- (i) $\tilde{\emptyset}, \tilde{X} \in \tau_\Delta$
 - (ii) The union of any member of binary soft sets in τ_Δ belongs to τ_Δ .
 - (iii) The intersection of any two binary soft sets in τ_Δ belongs to τ_Δ .
- Then $(U_1, U_2, \tau_\Delta, E)$ is called a binary soft topological space over U_1, U_2 .

Definition 10: Let (F, A) be any binary soft sub set of a binary soft topological space (X, Y, τ, E) then (F, A) is called

- 1) Binary soft pre-open set of (X, Y, τ, E) if $(F, A) \subseteq \text{in}(\text{cl}((F, A)))$ and
 - 2) Binary soft pre-closed set of (X, Y, τ, E) if $(F, A) \supseteq \text{cl}(\text{int}(F, A))$.
- The set of all binary P-open soft sets is denoted by BSPO (U) and the set of all binary P-closed sets is denoted by BSCO (U).

3. BINARY SOFT WEAK SEPARATION AXIOMS

In this section binary soft weak separation axioms in Binary Soft Topological Spaces are reflected.

Definition 11: A binary soft topological space $(\tilde{X}, \tilde{Y}, \mathcal{M}, A)$ is called a binary soft P_0 space if for any two binary soft points $(x_1, y_1), (x_2, y_2) \in \tilde{X}, \tilde{Y}$ such that $x_1 > x_2, y_1 > y_2$ there exists binary soft p-open sets (F_1, A) and (F_2, A) which behaves as $(x_1, y_1) \in (F_1, A), (x_2, y_2) \notin (F_1, A)$ or $(x_2, y_2) \in (F_2, A)$ and $(x_1, y_1) \notin (F_2, A)$.

Definition 12: A binary soft topological space $(\tilde{X}, \tilde{Y}, \mathcal{M}, A)$ is called a binary soft P_1 space if for any two binary soft points $(x_1, y_1), (x_2, y_2) \in \tilde{X}, \tilde{Y}$ such that $x_1 > x_2, y_1 > y_2$ If there exists binary soft p-open sets (F_1, A) and (F_2, A) which behaves as $(x_1, y_1) \in (F_1, A)$ and $(x_2, y_2) \notin (F_1, A)$ and $(x_2, y_2) \in (F_2, A)$ and $(x_1, y_1) \notin (F_2, A)$.

Definition 13: Two binary soft p-open sets $((F, A), (G, A))$ and $(H, A), (I, A)$ are said to be disjoint if $((F, A) \cap (H, A), (G, A) \cap (I, A)) = (\emptyset, \emptyset)$. That is $(F, A) \cap (H, A) = (\emptyset, \emptyset)$ and $(G, A) \cap (I, A) = (\emptyset, \emptyset)$.

Definition 14: A binary soft topological space $(\tilde{X}, \tilde{Y}, \mathcal{M}, A)$ is called a binary soft P_2 space if for any two binary soft points $(x_1, y_1), (x_2, y_2) \in \tilde{X}, \tilde{Y}$ such that $x_1 > x_2, y_1 > y_2$ If there exists binary soft p-open sets (F_1, A) and (F_2, A) which behaves as $(x_1, y_1) \in (F_1, A)$ and $(x_2, y_2) \in (F_2, A)$ and moreover (F_1, A) and (F_2, A) are disjoint.

Definition 15: A binary soft topological space $(\tilde{X}, \tilde{Y}, \tau \times \sigma, A)$ is called a binary soft P_0 with respect to the first coordinate if for every pair of binary points $(x_1, \alpha), (y_1, \alpha)$ there exists $((F, A), (G, A)) \in \tau \times \sigma$ with $x_1 \in (F, A), y_1 \notin (F, A), \alpha \in (G, A)$ where p-open (F, A) in τ and p-open (G, A) in σ .

Definition 16: A binary soft topological space $(\tilde{X}, \tilde{Y}, \tau \times \sigma, A)$ is called a binary soft P_0 with respect to the second coordinate if for every pair of binary points $(\beta, x_2), (\beta, y_2)$ there exists $((F, A), (G, A)) \in \tau \times \sigma$ with $\beta \in (F, A), x_2 \in (G, A), y_2 \notin (G, A)$ where p-open (F, A) in τ and p-open (G, A) in σ .

Definition 17: A binary soft topological space $(\tilde{X}, \tilde{Y}, \mathcal{M}, A)$ is called a binary soft P_0 space if for any two binary soft points $(e_{G_1}, e_{H_1}), (e_{G_2}, e_{H_2}) \in \tilde{X}, \tilde{Y}$ such that $e_{G_1} > e_{G_2}, e_{H_1} > e_{H_2}$ there exists binary soft p-open sets (F_1, A) and (F_2, A) which behaves as $(e_{G_1}, e_{H_1}) \in (F_1, A), (e_{G_2}, e_{H_2}) \notin (F_1, A)$ or $(e_{G_2}, e_{H_2}) \in (F_2, A)$ and $(e_{G_1}, e_{H_1}) \notin (F_2, A)$.

Definition 18: A binary soft topological space $(\tilde{X}, \tilde{Y}, \mathcal{M}, A)$ is called a binary soft P_1 space if for any two binary soft points $(e_{G_1}, e_{H_1}), (e_{G_2}, e_{H_2}) \in \tilde{X}, \tilde{Y}$ such that $e_{G_1} > e_{G_2}, e_{H_1} > e_{H_2}$ If there exists binary soft p-open sets (F_1, A) and (F_2, A) which behaves as $(e_{G_1}, e_{H_1}) \in (F_1, A)$ and $(e_{G_2}, e_{H_2}) \notin (F_1, A)$ and $(e_{G_2}, e_{H_2}) \in (F_2, A)$ and $(e_{G_1}, e_{H_1}) \notin (F_2, A)$.

Definition 19: A binary soft topological space $(\tilde{X}, \tilde{Y}, \mathcal{M}, A)$ is called a binary soft P_2 space if for any two binary soft points $(e_{G_1}, e_{H_1}), (e_{G_2}, e_{H_2}) \in \tilde{X}, \tilde{Y}$ such that $e_{G_1} > e_{G_2}, e_{H_1} > e_{H_2}$ If there exists binary soft p-open sets (F_1, A) and (F_2, A) which behaves as $(e_{G_1}, e_{H_1}) \in (F_1, A)$ and $(e_{G_2}, e_{H_2}) \in (F_2, A)$ and moreover

(F_1, A) and (F_2, A) are disjoint.

Definition 20: A binary soft topological space $(\tilde{X}, \tilde{Y}, \tau \times \sigma, A)$ is called a binary soft P_0 with respect to the first coordinate if for every pair of binary points $(e_{G_1}, \alpha), (e_{H_1}, \alpha)$ there exists $((F, A), (G, A)) \in \tau \times \sigma$ with $e_{G_1} \in (F, A), e_{H_1} \notin (F, A), \alpha \in (G, A)$ where p-open (F, A) in τ and p-open (G, A) in σ .

Definition 21: A binary soft topological space $(\tilde{X}, \tilde{Y}, \tau \times \sigma, A)$ is called a binary soft P_0 with respect to the second coordinate if for every pair of binary points $(\beta, e_{G_2}), (\beta, e_{H_2})$ there exists $((F, A), (G, A)) \in \tau \times \sigma$ with $\beta \in (F, A), e_{G_2} \in (G, A), e_{H_2} \notin (G, A)$ where p-open (F, A) in τ and p-open (G, A) in σ .

4. BINARY SOFT STRUCTURES WITH RESPECT TO ORDINARY POINTS

Theorem 1. If the binary soft topological space $(\tilde{X}, \tilde{Y}, \rho \times \sigma, A)$ is a binary soft P_0 , then (\tilde{X}, ρ, A) and (\tilde{Y}, σ, A) are soft P_0 .

Proof. We suppose $(\tilde{X}, \tilde{Y}, \rho \times \sigma, A)$ is a binary soft P_0 . Suppose $x_1, x_2 \in \tilde{X}$ and $y_1, y_2 \in \tilde{Y}$ with such that $x_1 > x_2, y_1 > y_2$. Since $(\tilde{X}, \tilde{Y}, \rho \times \sigma, A)$ is a binary soft P_0 , accordingly there binary soft p-open set $((F, A), (G, A))$ such that $(x_1, y_1) \in (F, A), (x_2, y_2) \notin (F, A)$ or $(x_2, y_2) \in (G, A)$ and $(x_1, y_1) \notin (G, A)$. This implies that either $x_1 \in (F, A), x_2 \notin (F, A), y_1 \in (G, A), y_2 \in (G, A)$ or $x_1 \in (F, A), x_2 \in (F, A), y_1 \in (G, A), y_2 \notin (G, A)$. This implies either $x_1 \in (F, A), x_2 \in (F, A)$ or $x_1 \in (F, A), x_2 \notin (F, A)$ and either $y_1 \in (G, A), y_2 \in (G, A)$ or $y_1 \in (G, A), y_2 \notin (G, A)$. Since $((F, A), (G, A)) \in \rho \times \sigma$, We have p-open (F, A) in ρ and p-open (G, A) in σ . this proves that (\tilde{X}, ρ, A) and (\tilde{Y}, σ, A) are soft P_0 .

Theorem 2. A binary soft topological space $(\tilde{X}, \tilde{Y}, \tau \times \sigma, A)$ is binary soft P_0 space with respect to first and the second coordinates, then $(\tilde{X}, \tilde{Y}, \tau \times \sigma, A)$ is binary soft P_0 space.

Proof. Let $(\tilde{X}, \tilde{Y}, \tau \times \sigma, A)$ is binary soft P_0 space with respect to first and the second coordinates. Let $(x_1, y_1), (x_2, y_2) \in \tilde{X} \times \tilde{Y}$ with $x_1 > x_2, y_1 > y_2$. Take $\alpha \in \tilde{Y}$ and $\beta \in \tilde{X}$. Then $(x_1, \alpha), (x_2, \alpha) \in \tilde{X} \times \tilde{Y}$. Since $(\tilde{X}, \tilde{Y}, \tau \times \sigma, A)$ is a binary soft P_0 space with respect to the first coordinate, by using definition, there exists $((F, A), (G, A)) \in \tau \times \sigma$ with $x_1 \in (F, A), x_2 \notin (F, A), \alpha \in (G, A)$. Since $(\beta, y_1), (\beta, y_2) \in \tilde{X} \times \tilde{Y}$, by using the arguments and using definition there exists $((H, A), (K, A)) \in \tau \times \sigma$ with $y_1 \in (K, A), y_2 \notin (K, A), \beta \in (H, A)$. Therefore, $(x_1, y_1) \in (F, A), (K, A)$ and $(x_2, y_2) \notin (F, A), (K, A)$. Hence $(\tilde{X}, \tilde{Y}, \tau \times \sigma, A)$ is called a binary soft b- T_0 .

Theorem 3. A binary soft topological space (\tilde{X}, τ, A) and (\tilde{Y}, σ, A) are soft P_1 spaces if and only if the binary soft topological space $(\tilde{X}, \tilde{Y}, \tau \times \sigma, A)$ is soft binary P_1 .

Proof. Suppose (\tilde{X}, τ, A) and (\tilde{Y}, σ, A) are soft P_1 spaces. Let $(x_1, y_1), (x_2, y_2) \in \tilde{X} \times \tilde{Y}$ with $x_1 > x_2, y_1 > y_2$. since (\tilde{X}, τ, A) is soft P_1 space, there exists soft p-open sets such that $(F, A), (G, A) \in \tau, x_1 \in (F, A)$ and $x_2 \notin (G, A)$ such that $x_1 \in (F, A)$ and $x_2 \notin (F, A)$. Also, since (\tilde{Y}, σ, A) is soft P_1 space, there exists soft p-open sets such that $(H, A), (I, A) \in \sigma, y_1 \in (H, A)$ and $y_2 \notin (I, A)$ such that $y_1 \in (H, A)$ and $y_2 \notin (H, A)$. thus $(x_1, y_1) \in (F, A), (H, A)$ and $(x_2, y_2) \notin (G, A), (I, A)$ with $(x_1, y_1) \in (G, A), (I, A)$ and $(x_1, y_1) \in (F, A), (H, A)$. This implies that $(\tilde{X}, \tilde{Y}, \tau \times \sigma, A)$ is soft binary P_1 . Conversely assume that $(\tilde{X}, \tilde{Y}, \tau \times \sigma, A)$ is soft binary P_1 . Let $x_1, x_2 \in \tilde{X}$ and $y_1, y_2 \in \tilde{Y}$ such that $x_1 > x_2, y_1 > y_2$. Therefore $(x_1, y_1), (x_2, y_2) \in \tilde{X} \times \tilde{Y}$. Since $(\tilde{X}, \tilde{Y}, \tau \times \sigma, A)$ is soft binary P_1 , there exists $(F, A), (G, A)$ and $(H, A), (I, A) \in \tau \times \sigma$ such that $(x_1, y_1) \in (F, A), (G, A)$ and $(x_1, y_1) \in (H, A), (I, A)$ such that $(x_1, y_1) \in (H, A), (I, A)$ and $(x_2, y_2) \notin (F, A), (G, A)$. Therefore, $x_1 \in (F, A), x_2 \notin (H, A)$ and $x_1 \in (H, A)$ and $x_2 \notin (F, A)$ and $y_1 \in (G, A)$ and $y_1 \in (I, A)$ and $y_2 \notin (I, A)$ and $y_2 \notin (G, A)$. Since $(F, A), (G, A) \in \tau \times \sigma$, We have $(F, A), (H, A) \in \tau$ and $(G, A), (I, A) \in \sigma$. This proves that (\tilde{X}, τ, A) and (\tilde{Y}, σ, A) are soft P_1 spaces.

Theorem 4. A binary soft topological space $(\tilde{X}, \tilde{Y}, \mathcal{M}, A)$ is binary soft P_1 space if and only if every binary soft point $\wp(X) \times \wp(Y)$ is binary soft p-closed.

Proof. Suppose that $(\tilde{X}, \tilde{Y}, \mathcal{M}, A)$ is binary soft P_1 space. Let $(x, y) \in \tilde{X} \times \tilde{Y}$. Let $(\{x\}, \{y\}) \in \wp(X) \times \wp(Y)$. We shall show that $(\{x\}, \{y\})$ is binary soft p-closed. it is sufficient to show that $(X \setminus \{x\}, Y \setminus \{y\})$ is binary soft p-open. Let $(a, b) \in (X \setminus \{x\}, Y \setminus \{y\})$. This implies that $a \notin X \setminus \{x\}$ and $b \notin Y \setminus \{y\}$. hence $a =$

x and $b \neq y$. That is, (a, b) and (x, y) are distinct binary soft points of $X \times Y$. Since $(\tilde{X}, \tilde{Y}, \mathcal{M}, A)$ is binary soft P_1 space, there exists binary soft p-open sets $((F, A), (G, A))$ and $(H, A), (I, A)$ such that $(a, b) \in ((F, A), (G, A))$ and $(x, y) \in ((H, A), (I, A))$ such that $(a, b) \in ((F, A), (G, A))$ and $(x, y) \in ((F, A), (G, A)) \subseteq (\{x\}^c, \{y\}^c)$. Hence $(\{x\}^c, \{y\}^c)$ is a soft neighbourhood of (a, b) . This implies that $(\{x\}, \{y\})$ is binary soft p-closed. Conversely, suppose that $(\{x\}, \{y\})$ is binary soft b-closed for every $(x, y) \in X \times Y$. Suppose $(x_1, y_1), (x_2, y_2) \in X \times Y$ with $x_1 > x_2, y_1 > y_2$. Therefore, $(x_2, y_2) \in (\{x_1\}^c, \{y_1\}^c)$ and $\mathcal{E}(\{x_1\}^c, \{y_1\}^c)$ is binary soft p-open. Also $(x_1, y_1) \in (\{x_2\}^c, \{y_2\}^c)$ and $\mathcal{E}(\{x_2\}^c, \{y_2\}^c)$ is binary soft p-open set. Also $(x_1, y_1) \in (\{x_2\}^c, \{y_2\}^c)$ and $\mathcal{E}(\{x_2\}^c, \{y_2\}^c)$ is binary soft p-open set. This shows that $(\tilde{X}, \tilde{Y}, \mathcal{M}, A)$ is binary soft P_1 space.

Theorem 5. A binary soft topological space (\tilde{X}, τ, A) and (\tilde{Y}, σ, A) are soft P_2 spaces if and only if the binary soft topological space $(\tilde{X}, \tilde{Y}, \tau \times \sigma, A)$ is soft binary P_2 .

Proof. Suppose (\tilde{X}, τ, A) and (\tilde{X}, σ, A) are soft P_2 spaces. Let $(x_1, y_1), (x_2, y_2) \in X \times Y$ with $x_1 > x_2, y_1 > y_2$. Since (\tilde{X}, τ, A) is soft P_2 space, there exists soft p-open sets such that $(F, A), (G, A) \in \tau, x_1 \in (F, A)$ and $x_2 \in (G, A)$ such that $x_1 \notin (G, A)$ and $x_2 \notin (F, A)$. Also, since (\tilde{Y}, σ, A) is soft P_2 space, there exists disjoint soft p-open sets such that $(H, A), (I, A) \in \sigma, y_1 \in (H, A)$ and $y_2 \in (I, A)$ such that $y_1 \notin (I, A)$ and $y_2 \notin (H, A)$. Thus $(x_1, y_1) \in ((F, A), (H, A))$ and $(x_2, y_2) \in ((G, A), (I, A))$ with $(x_1, y_1) \in ((G, A), (I, A))$ and $(x_2, y_2) \in ((F, A), (H, A))$. Since (F, A) and (G, A) are disjoint, $(F, A) \cap (H, A) = (\Phi, \Phi)$. Also since $(H, A) \cap (I, A) = (\Phi, \Phi)$. Thus $(F, A) \cap (H, A), (G, A) \cap (I, A) = (\Phi, \Phi)$. This implies that we have this implies that $(\tilde{X}, \tilde{Y}, \tau \times \sigma, A)$ is soft binary P_2 . Conversely assume that $(\tilde{X}, \tilde{Y}, \tau \times \sigma, A)$ is soft binary P_2 . Let $x_1, x_2 \in X$ and $y_1, y_2 \in Y$ such that $x_1 > x_2, y_1 > y_2$. Therefore $(x_1, y_1), (x_2, y_2) \in X \times Y$. Since $(\tilde{X}, \tilde{Y}, \tau \times \sigma, A)$ is soft binary P_2 , there exists binary soft p-open sets $((F, A), (G, A))$ and $((H, A), (I, A))$ such that $(x_1, y_1) \in ((F, A), (G, A))$ and $(x_2, y_2) \in ((H, A), (I, A))$ such that $(x_1, y_1) \in ((H, A), (I, A))$ and $(x_2, y_2) \in ((F, A), (G, A))$. Therefore, $x_1 \in (F, A), x_2 \in (H, A)$ and $x_1 \in (H, A)$ and $x_2 \in (F, A)$ and $y_1 \in (G, A)$ and $y_2 \in (I, A)$ and $y_1 \in (I, A)$ and $y_2 \in (G, A)$. Since $(F, A), (G, A) \in \tau \times \sigma$, we have $(F, A), (H, A) \in \tau$ and $(G, A), (I, A) \in \sigma$. This proves that (\tilde{X}, τ, A) and (\tilde{X}, σ, A) are soft P_2 spaces.

5. BINARY SOFT STRUCTURES WITH RESPECT TO SOFT POINTS

Theorem 6. If the binary soft topological space $(\tilde{X}, \tilde{Y}, \rho \times \sigma, A)$ is a binary soft P_0 , then (\tilde{X}, ρ, A) and (\tilde{Y}, σ, A) are soft P_0 .

Proof. We suppose $(\tilde{X}_A, \tilde{Y}_A, \rho \times \sigma, A)$ is a binary soft P_0 . Suppose $e_{G_1}, e_{G_2} \in \tilde{X}_A$ and $e_{H_1}, e_{H_2} \in \tilde{Y}_A$ with such that $e_{G_1} > e_{G_2}, e_{H_1} > e_{H_2}$. Since $(\tilde{X}_A, \tilde{Y}_A, \rho \times \sigma, A)$ is a binary soft P_0 , accordingly there binary soft p-open set $((F, A), (G, A))$ such that $(e_{G_1}, e_{H_1}) \in ((F, A), (G, A))$; $(e_{G_2}, e_{H_2}) \in ((F, A), (G, A))$ or $(e_{G_1}, e_{H_1}) \in ((F, A), (G, A))$; $(e_{G_2}, e_{H_2}) \in ((F, A), (G, A))$. This implies that either $e_{G_1} \in (F, A)$; $e_{G_2} \in (F, A)$; $e_{H_1} \in (G, A)$; $e_{H_2} \in (G, A)$; or $e_{G_1} \in (F, A)$; $e_{H_1} \in (G, A)$; $e_{H_2} \in (G, A)$. This implies either $e_{G_1} \in (F, A)$; $e_{G_2} \in (F, A)$ or $e_{G_1} \in (F, A)$; $x_1 \in (F, A)$ and either $e_{H_1} \in (G, A)$; $e_{H_2} \in (G, A)$ or $e_{H_1} \in (G, A)$; $e_{H_2} \in (G, A)$. Since $((F, A), (G, A)) \in \rho \times \sigma$, we have p-open $(F, A) \in \rho$ and p-open $(G, A) \in \sigma$. This proves that (\tilde{X}, ρ, A) and (\tilde{Y}, σ, A) are soft P_0 .

Theorem 7. A binary soft topological space $(\tilde{X}, \tilde{Y}, \tau \times \sigma, A)$ is binary soft P_0 space with respect to first and the second coordinates, then $(\tilde{X}, \tilde{Y}, \tau \times \sigma, A)$ is binary soft P_0 space.

Proof. Let $(\tilde{X}, \tilde{Y}, \tau \times \sigma, A)$ is binary soft P_0 space with respect to first and the second coordinates. Let $(e_{G_1}, e_{H_1}), (e_{G_2}, e_{H_2}) \in X \times Y$ with $e_{G_1} > e_{G_2}, e_{H_1} > e_{H_2}$. Take $\alpha \in \tilde{Y}$ and $\beta \in \tilde{X}$. Then $(e_{G_1}, \alpha), (e_{G_2}, \alpha) \in X \times Y$. Since $(\tilde{X}, \tilde{Y}, \tau \times \sigma, A)$ is a binary soft P_0 space with respect to the first coordinate, by using definition, there exists $((F, A), (G, A)) \in \tau \times \sigma$ with $e_{G_1} \in (F, A)$, $e_{G_2} \in (F, A)$, $\alpha \in (G, A)$. Since $(\beta, e_{H_1}), (\beta, e_{H_2}) \in X \times Y$, by using the arguments and using definition there exists $((H, A), (K, A)) \in \tau \times \sigma$ with $e_{H_1} \in (H, A)$, $e_{H_2} \in (H, A)$, $\beta \in (K, A)$ where (H, A) is p-open in τ and (K, A) is p-open in σ . Therefore, $(e_{G_1}, e_{H_1}) \in ((F, A), (K, A))$ and $(e_{G_2}, e_{H_2}) \in ((F, A), (K, A))$. Hence $(\tilde{X}, \tilde{Y}, \tau \times \sigma, A)$ is called a binary soft P_0 .

Theorem 8. A binary soft topological space (\tilde{X}, τ, A) and (\tilde{Y}, σ, A) are soft P_1 spaces if and only if the binary soft topological space $(\tilde{X}, \tilde{Y}, \tau \times \sigma, A)$ is

soft binary P_1 .

Proof. Suppose (\tilde{X}, τ, A) and (\tilde{X}, σ, A) are soft P_1 spaces. Let $(e_{G_1}, e_{H_1}), (e_{G_2}, e_{H_2}) \in X \times Y$ with $e_{G_1} > e_{G_2}, e_{H_1} > e_{H_2}$. Since (\tilde{X}, τ, A) is soft P_1 space, there exists soft p-open sets such that $(F, A), (G, A) \in \tau, e_{G_1} \in (F, A)$ and $e_{G_2} \in (G, A)$ such that $e_{G_1} \notin (G, A)$ and $e_{G_2} \notin (F, A)$. Also, since (\tilde{Y}, σ, A) is soft P_1 space, there exists soft p-open sets such that $(H, A), (I, A) \in \sigma, e_{H_1} \in (H, A)$ and $e_{H_2} \in (I, A)$ such that $e_{H_1} \notin (I, A)$ and $e_{H_2} \notin (H, A)$. Thus $(e_{G_1}, e_{H_1}) \in ((F, A), (H, A))$ and $(e_{G_2}, e_{H_2}) \in ((G, A), (I, A))$ with $(e_{G_1}, e_{H_1}) \in ((G, A), (I, A))$ and $(e_{G_2}, e_{H_2}) \in ((F, A), (H, A))$. This implies that $(\tilde{X}, \tilde{Y}, \tau \times \sigma, A)$ is soft binary P_1 . Conversely assume that $(\tilde{X}, \tilde{Y}, \tau \times \sigma, A)$ is soft binary P_1 . Let $e_{G_1}, e_{G_2} \in X$ and $e_{H_1}, e_{H_2} \in Y$ such that $e_{G_1} > e_{G_2}, e_{H_1} > e_{H_2}$. Therefore $(e_{G_1}, e_{H_1}), (e_{G_2}, e_{H_2}) \in X \times Y$. Since $(\tilde{X}, \tilde{Y}, \tau \times \sigma, A)$ is soft binary P_1 , there exists p-open sets $(F, A), (G, A)$ and p-open sets $(H, A), (I, A) \in \tau \times \sigma, (e_{G_1}, e_{H_1}) \in ((F, A), (G, A))$ and $(e_{G_2}, e_{H_2}) \in ((H, A), (I, A))$ such that $(e_{G_1}, e_{H_1}) \in ((H, A), (I, A))$ and $(e_{G_2}, e_{H_2}) \in ((F, A), (G, A))$. Therefore, $e_{G_1} \in (F, A), e_{G_2} \in (H, A)$ and $e_{G_1} \in (H, A)$ and $e_{G_2} \in (F, A)$ and $e_{H_1} \in (G, A)$ and $e_{H_2} \in (I, A)$ and $e_{H_1} \in (I, A)$ and $e_{H_2} \in (G, A)$. Since $(F, A), (G, A) \in \tau \times \sigma$, we have $(F, A), (H, A) \in \tau$ and $(G, A), (I, A) \in \sigma$. This proves that (\tilde{X}, τ, A) and (\tilde{X}, σ, A) are soft P_1 spaces.

Theorem 9. A binary soft topological space $(\tilde{X}, \tilde{Y}, \mathcal{M}, A)$ is binary soft P_1 space if and only if every binary soft point $\wp(X) \times \wp(Y)$ is binary soft p-closed.

Proof. Suppose that $(\tilde{X}, \tilde{Y}, \mathcal{M}, A)$ is binary soft P_1 space. Let $(x, y) \in X \times Y$. Let $(\{x\}, \{e_{H_1}\}) \in \tilde{\mathcal{E}}\wp(X) \times \wp(Y)$. We shall show that $(\{x\}, \{e_{H_1}\})$ is binary soft p-closed. It is sufficient to show that $(X \setminus \{e_{G_1}\}, Y \setminus \{e_{H_1}\})$ is binary soft p-open. Let $(a, b) \in (X \setminus \{e_{G_1}\}, Y \setminus \{e_{H_1}\})$. This implies that $a \in X \setminus \{e_{G_1}\}$ and $b \in Y \setminus \{e_{H_1}\}$, hence $a \neq e_{G_1}$ and $b \neq e_{H_1}$. That is, (a, b) and (e_{G_1}, e_{H_1}) are distinct binary soft points of $X \times Y$. Since $(\tilde{X}, \tilde{Y}, \mathcal{M}, A)$ is binary soft P_1 space, there exists binary soft p-open sets $((F, A), (G, A))$ and $(H, A), (I, A)$ such that $(a, b) \in ((F, A), (G, A))$ and $(x, y) \in ((H, A), (I, A))$ such that $(a, b) \in ((H, A), (I, A))$ and $(e_{G_1}, e_{H_1}) \in ((F, A), (G, A))$. Therefore, $((F, A), (G, A)) \subseteq (\{e_{G_1}\}^c, \{e_{H_1}\}^c)$. Hence $(\{e_{G_1}\}^c, \{e_{H_1}\}^c)$ is a soft neighbourhood of (a, b) . This implies that $(\{e_{G_1}\}, \{e_{H_1}\})$ is binary soft p-closed. Conversely, suppose that $(\{e_{G_1}\}, \{e_{H_1}\})$ is binary soft p-closed for every $(e_{G_1}, e_{H_1}) \in X \times Y$. Suppose $(e_{G_1}, e_{H_1}), (e_{G_2}, e_{H_2}) \in X \times Y$ with $e_{G_1} > e_{G_2}, e_{H_1} > e_{H_2}$. Therefore, $(e_{G_2}, e_{H_2}) \in (\{e_{G_1}\}^c, \{e_{H_1}\}^c)$ and $\mathcal{E}(\{e_{G_1}\}^c, \{e_{H_1}\}^c)$ is binary soft p-open. Also $(e_{G_1}, e_{H_1}) \in (\{e_{G_2}\}^c, \{e_{H_2}\}^c)$ and $\mathcal{E}(\{e_{G_2}\}^c, \{e_{H_2}\}^c)$ is binary soft p-open set. Also $(e_{G_1}, e_{H_1}) \in (\{e_{G_2}\}^c, \{e_{H_2}\}^c)$ and $\mathcal{E}(\{e_{G_2}\}^c, \{e_{H_2}\}^c)$ is binary soft p-open set. This shows that $(\tilde{X}, \tilde{Y}, \mathcal{M}, A)$ is binary soft P_1 space.

Theorem 10. A binary soft topological space (\tilde{X}, τ, A) and (\tilde{Y}, σ, A) are soft P_2 spaces if and only if the binary soft topological space $(\tilde{X}, \tilde{Y}, \tau \times \sigma, A)$ is soft binary P_2 .

Proof. Suppose (\tilde{X}, τ, A) and (\tilde{X}, σ, A) are soft P_2 spaces. Let $(e_{G_1}, e_{H_1}), (e_{G_2}, e_{H_2}) \in X \times Y$ with $e_{G_1} > e_{G_2}, e_{H_1} > e_{H_2}$. Since (\tilde{X}, τ, A) is soft P_2 space, there exists soft p-open sets such that $(F, A), (G, A) \in \tau, e_{G_1} \in (F, A)$ and $e_{G_2} \in (G, A)$ such that $e_{G_1} \notin (G, A)$ and $e_{G_2} \notin (F, A)$. Also, since (\tilde{Y}, σ, A) is soft P_2 space, there exists disjoint soft p-open sets such that $(H, A), (I, A) \in \sigma, e_{H_1} \in (H, A)$ and $e_{H_2} \in (I, A)$ such that $e_{H_1} \notin (I, A)$ and $e_{H_2} \notin (H, A)$. Thus $(e_{G_1}, e_{H_1}) \in ((F, A), (H, A))$ and $(e_{G_2}, e_{H_2}) \in ((G, A), (I, A))$ with $(e_{G_1}, e_{H_1}) \in ((G, A), (I, A))$ and $(e_{G_2}, e_{H_2}) \in ((F, A), (H, A))$. Since (F, A) and (G, A) are disjoint, $(F, A) \cap (H, A) = (\Phi, \Phi)$. Also since $(H, A) \cap (I, A) = (\Phi, \Phi)$. Thus $(F, A) \cap (H, A), (G, A) \cap (I, A) = (\Phi, \Phi)$. This implies that we have this implies that $(\tilde{X}, \tilde{Y}, \tau \times \sigma, A)$ is soft binary P_2 . Conversely assume that $(\tilde{X}, \tilde{Y}, \tau \times \sigma, A)$ is soft binary P_2 . Let $e_{G_1}, e_{G_2} \in X$ and $e_{H_1}, e_{H_2} \in Y$ such that $e_{G_1} > e_{G_2}, e_{H_1} > e_{H_2}$. Therefore $(e_{G_1}, e_{H_1}), (e_{G_2}, e_{H_2}) \in X \times Y$. Since $(\tilde{X}, \tilde{Y}, \tau \times \sigma, A)$ is soft binary P_2 , there exists binary soft p-open sets $((F, A), (G, A))$ and $((H, A), (I, A))$ such that $(e_{G_1}, e_{H_1}) \in ((F, A), (G, A))$ and $(e_{G_2}, e_{H_2}) \in ((H, A), (I, A))$ such that $(e_{G_1}, e_{H_1}) \in ((H, A), (I, A))$ and $(e_{G_2}, e_{H_2}) \in ((F, A), (G, A))$. Therefore, $e_{G_1} \in (F, A), e_{G_2} \in (H, A)$ and $e_{G_1} \in (H, A)$ and $e_{G_2} \in (F, A)$ and $e_{H_1} \in (G, A)$ and $e_{H_2} \in (I, A)$ and $e_{H_1} \in (I, A)$ and $e_{H_2} \in (G, A)$. Since $(F, A), (G, A) \in \tau \times \sigma$, we have $(F, A), (H, A) \in \tau$ and $(G, A), (I, A) \in \sigma$. This proves that (\tilde{X}, τ, A) and (\tilde{X}, σ, A) are soft P_2 spaces.

6. CONCLUSION

The soft Separation Axioms namely, T_0 , T_1 - T_2 are extended to binary soft P - T_0 , P - T_1 and P - T_2 structures with respect to first and second coordinates.

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