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## SUPRA SOFT R-SEPARATION AXIOMS

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### ARTICLE DETAILS

### **ABSTRACT**

### Article History:

Received 26 June 2018 Accepted 2 July 2018 Available online 1 August 2018 This paper introduces the notion of supra soft b-separation axioms based on the supra b-open soft sets which are more general than supra open soft sets. We investigate the relationships between these supra soft separation axioms. Furthermore, with the help of examples it is established that the converse does not hold.

### **KEYWORDS**

Soft sets, Soft topological spaces, Supra soft topological spaces, Supra b-open soft sets, Supra soft b-T<sub>i</sub> spaces (i=1, 2, 3, 4), Supra soft continuity, Supra b-irresolute open soft function.

### 1. INTRODUCTION

The real world is too complex for our immediate and direct understanding. Several models of reality that are simplifications of aspects of the real word have been established. Unfortunately, these mathematical models are too complicated, and we cannot find the exact solutions. The uncertainty of data while modeling the problems in engineering, physics, computer sciences, economics, social sciences, medical sciences and many other diverse fields makes it unsuccessful to use the traditional classical methods. These may be due to the uncertainties of natural environmental phenomena, of human knowledge about the real world or to the limitations of the means used to measure objects. Thus, classical set theory, which is based on the crisp and exact case, may not be fully suitable for handling such problems of uncertainty. There are several theories, for example, theory of fuzzy sets, theory of intuitionistic fuzzy sets, theory of vague sets, theory of interval mathematics and theory of rough sets [1-4]. These can be considered as tools for dealing with uncertainties, but all these theories have their own In recent years, many interesting applications of soft set theory have been expanded by embedding the ideas of fuzzy sets [5-11]. To develop soft set theory, the operations of the soft sets are redefined and a uni-int decision making method was constructed by using these new operations [12]. It got some stability only after the introduction of soft topology in 2011 [13]. In a study, scientist introduced some soft operations such as semi open soft, pre-open soft,  $\alpha$  — open soft and  $\beta$  –open soft and investigated their properties in detail. the notion of pre open soft sets is extended in [14, 15]. In other research, some researchers introduced the notion of soft semi separation axioms. In particular they study the properties of the soft semi regular spaces and soft semi normal spaces [16]. The notion of soft ideal was initiated for the first time by Kandil et al. [17].

They also introduced the concept of soft local function. These concepts are discussed with a view to find new soft topologies from the original one, called soft topological spaces with soft ideal (X, \tau, E, I). Applications to various fields were further investigated [18-24]. The notion of supra soft topological spaces was initiated for the first time [25]. Recently, in study they introduced the concept of soft supra g-closed soft sets in supra soft topological spaces, which is generalized in [23, 26].

The notion of b-open soft sets was initiated for the first time, which is generalized to the supra soft topological spaces in [27-29]. Properties of b-open soft sets in are discussed [30]. The main purpose of this paper is to generalize the notion of supra soft separation axioms by using the notions of supra b-open soft sets difficulties [31].

The reason for these difficulties is, possibly, the inadequacy of the parameterization tool of the theory as it was mentioned by Molodtsov in [32]. He initiated the concept of soft set theory as a new mathematical tool which is free from the problems mentioned above. In his paper, he presented the fundamental results of the new theory and successfully applied it to several directions such as smoothness of functions, game theory, operations research, Riemann-integration, Perron integration, theory of probability etc [32]. A soft set is a collection of approximate descriptions of an object. He also showed how soft set theory is free from the parameterization inadequacy syndrome of fuzzy set theory, rough set theory, probability theory and game theory. Soft systems provide a very general framework with the involvement of parameters. Research works on soft set theory and its applications in various fields are progressing rapidly. After presentation of the operations of soft sets the properties and applications of soft set theory have been studied increasingly [9, 11, 33,34].

### 2. PRELIMINARIES

**<u>Definition 2.1:</u>** [32]. Let X is an initial universe and E is a set of parameters. Let P(X) denote the power set of X. A pair (F,E) denoted by F is called a soft set over X, where F is a mapping F:  $E \rightarrow P(X)$  given by. In other words, a soft set over X is a parameterized family of subsets of the universe X. For  $e \in E, F(e)$ , may be considered as the set of e-approximate elements of the soft set (F,E) I .e (F,E)= $\{(e, F(e) : e \in E, F: E \rightarrow P(X))\}\$ if  $e \notin E$  then  $F(e)=\emptyset$  The set of all soft sets over X will be denoted by  $S_E(X)$ .

**Definition 2.2:** [35]. Let  $F, G \in S_E(X)$  Then,

- (1)*F* is said to be null soft set, denoted by  $\widehat{\emptyset}$ , if F(e)= $\emptyset$  for all e∈ E. (2) F is said to be absolute soft set, denoted by  $\hat{X}$ , if F(e) = X for all  $e \in E$ (3) F is soft subset of G, denoted by  $F \subseteq G$ , if  $F(e) \subseteq G(e)$  for all  $e \in E$ (4) F and G are soft equal, denoted by F = G, if  $F \subseteq G$ , and  $G \subseteq F$ ,
- (5) The soft union of F and G, denoted by  $F \cup G$ , is a soft set over X and

FU G: E  $\rightarrow$ P(X) such that (FU G)(e)= F(e) U G(e) for all e  $\in$  E.

- (6) The soft intersection of F and G, denoted by  $F \cap G$ , is a soft set over X and defined by  $F \cap G$ :  $E \rightarrow P(X)$  such that  $(F \cap G)(e) = F(e)G(e)$  for all  $e \in E$ .
- (7) The soft complement  $(\widehat{X} F)$  of a soft set F is denoted by  $F^C$  and defined by  $F^C$ :  $E \to P(X)$  such that  $F^C(e) = X/F(e)$   $e \in E$  for all.

**<u>Definition 2.3:</u>** [36]. A soft set  $F \in S_E(X)$  is called a soft point, denoted by  $e_F$ , If there exist an  $e \in E$  such that  $F(e) \neq \emptyset$  and  $F(e) = \emptyset$  for each  $e \in E$ . The soft point  $e_F$  is said to be in the soft set  $e_F$ , if  $e_F$  is and we write  $e_F \in E$ .

**<u>Definition 2.4:</u>** [16,13]. A soft set *F* over *X* where  $F(e) = \{x\}$ ,  $\forall e \in E$  called singleton soft point is and denoted by  $X_E$  or (x, E)

**Definition 2.5:** [13]. Let  $\tau$  be a collection of soft sets over a universe X with a fixed set of parameters E, then  $\tau \subseteq S_E(X)$  is called a soft topology on if

- (1)  $\emptyset$ , X belong to  $\tau$
- (2) The union of any number of soft sets in  $\tau$  belongs to  $\tau$
- (3) The intersection of any two soft sets in  $\tau$  belong to  $\tau$

The triplet  $(X, \tau, E)$  is called a soft topological space. A soft set F over X is said to be an open soft set in X if  $F \in \tau$ , and it is called a closed soft set in X, if its relative complement  $F^C$  is a soft open set and we write F is a soft  $\tau$ -closed

**<u>Definition 2.6:</u>** [13]. Let  $(X, \tau, E)$  be a soft topological space over X and  $F \in S_E(X)$ .

Then the soft interior and soft closure of F, denoted by int(F) and cl(F), respectively, are

Defined as:  $Int(F)=\cup\{G/G \text{ is a soft open set and } G\subseteq F\}$  ,  $Cl(F)=\cap\{H/H \text{ is a soft cloused set and } F\subseteq H\}$ 

**<u>Definition 2.7:</u>** [37]. Let  $S_E(X)$  and  $S_E(Y)$  be families of soft sets, $\mu: X \to Y$  and  $P: E \to K$  be mappings. Therefore  $f_{\mu P}: S_E(X) \to S_E(Y)$  is called a soft function.

(1)If  $F \in S_E(X)$ , then the image of F under  $f_{\mu P}$ , written  $asf_{\mu P}(F)$ , is a soft set

in 
$$S_K(Y)$$
 such that  $f_{P\mu}(F)(K) = \begin{cases} \bigcup_{e \in P^{-1}(k)\mu(F(e))}, P^{-1}(k) \neq \emptyset \\ \emptyset, \text{ otherwaies} \end{cases}$ 
For each  $k \in K$ 

 $(2) \ \text{If } G \in \ S_K(Y), \text{ then the inverse image of } G \text{ under } f_{\mathbb{P}\mu} \text{ , written a, } f^{-1}_{\mathbb{P}\mu} \text{ is a soft set in } S_E(X) \text{ such that } f^{-1}_{\mathbb{P}\mu} (G)(e) = \{(\mu^{-1} \Big( G \Big( P(e) \Big) \Big), p(e) \in Y \}$ 

For each the soft function  $f_{P\mu}$  is called surjective if P and  $\mu$  are surjective, also is said to be injective if P and  $\mu$  are injective. Several properties and characteristics  $f_{P\mu}$  and  $f^{-1}_{P\mu}$  are reported in detail in [37].

**<u>Definition 2.8:</u>** [36]. Let  $(X,\tau_1,E)$  and  $(Y,\tau_2,E)$  be soft topological spaces and be a function

 $f_{P\mu}:\,S_E(X)\rightarrow\,S_E(Y).$  Then, the function  $f_{P\mu}\,$  is called

- (1) Continuous soft if  $f^{-1}_{\mathbb{P}\mu}(G) \in \tau_1$  for each  $G \in \tau_2$ .
- (2) Open soft if  $f_{P\mu}(F) \in \tau_2$  for each  $F \in \tau_1$ .

**<u>Definition 2.9:</u>** [25]. Let  $\tau$  be a collection of soft sets over a universe with a fixed set of parameters E , then  $\mu \subseteq \tau$  is called supra soft topology on X with a fixed set if

- (1)  $X, \emptyset \in \mu$
- (2) The soft union of any number of soft sets in  $\mu$  belongs to  $\mu$  The triplet  $(X, \mu, E)$  is called supra soft topological space (or supra soft spaces) over X.

**<u>Definition 2.10:</u>** [25]. Let  $(X, \tau, E)$  be a soft topological space and  $(X, \mu E)$  be a supra soft topological space. We say that,  $\mu$  is a supra soft topology associated with  $\tau$  if  $\tau \subseteq \mu$ 

**<u>Definition 2.11:</u>** [25]. Let  $(X, \mu E)$  be a supra soft topological space over X, then the members of  $\mu$  are said to be supra open soft sets in X.

**<u>Definition 2.12:</u>** [25]. Let  $(X, \mu, E)$  be a supra soft topological space over X. Then, the supra soft  $\operatorname{int}^S(F)$  interior and supra soft closure of  $\operatorname{cl}^S(F)$ , denoted by and , Respectively, are defined respectively, are defined as  $\operatorname{int}^S(F) = \bigcup \{G/G \text{ is a supra open set and } G \subseteq F\}$ ,  $\operatorname{cl}^S(F) = \bigcap \{H/H \text{ is a soft closed} \text{ set and } F \subseteq H\}$  Clearly  $\operatorname{int}^S(F)$  is the largest supra open soft set

over X which contained in F and  $cl^S(F)\,$  is the smallest supra closed soft set over X which contains F.

**<u>Definition 2.13</u>**: A soft set (A, E) in a supra soft topological space  $(X, \mu, E)$  be will be termed soft supra regular open set denoted as S, S, R, O(X) if and only if there exists a soft open set (F, E) = int(cl(F, E)) and soft regular closed set if set (F, E) = cl(in(F, E)) denoted by as S, S, R, C(X) in short hand.

**<u>Definition 2.14:</u>** Let  $(X, \mu E)$  be a supra soft topological space over X and  $F \in S_E(X)$ .

Then, the supra soft regular soft interior and supra soft regular closure of X, denoted by  $\operatorname{int}^S(F)$  and  $\operatorname{cl}^S(F)$ , respectively, are defined as  $\operatorname{int}_B^S(F) = \bigcup \{G/G \text{ is a supra soft regular open set and } G \subseteq F\}$ 

,  $cl_B^S(F) = \bigcap \{H/H \text{ is a supra soft regular cloused set and } F \subseteq H\}$ 

**<u>Definition 2.15:</u>** [25,31]. Let  $(X,\tau_1,E)$  and  $(Y,\tau_2,E)$  be soft topological spaces,  $\mu_1$  and  $\mu_2$  be associated supra soft topologies with  $,\tau_1$  and  $,\tau_2$ , respectively. The soft function

 $f_{P\mu}: S_E(X) \rightarrow S_E(Y)$ . is called

- (1) Supra continuous soft function if  $f^{-1}{}_{\mathbb{P}\mu}$  (F)  $\in \mu_1$  for each Fé  $\tau_2$
- (2) Supra open soft if  $f_{\mathbb{P}\mu}(F) \in \mu_1$  for  $F \in \tau_1$ .
- (3) Supra irresolute soft if  $f^{-1}_{P\mu}(F) \in \mu_1$  for each  $F \in \mu_2$
- (4) Supra irresolute open soft if  $f_{P\mu}(F) \in \mu_2$  for each  $F \in \mu_1$ .
- (5) Supra b-continuous soft if  $f^{-1}_{P\mu}(F) \in SBOS(X, \mu_1, E)$  for each  $F \in \mu_2$ .

### 3. SUPRA SOFT R-SEPARATION AXIOMS

**Definition 3.1:** Let  $(X,\tau,E)$  be a soft topological space and  $\mu$  be an associated supra soft topology. Let  $x,y\in Y$  such that x< y or x> y. Then,  $(X,\mu,E)$  is called

- (1) Supra soft  $R_0$ -space if there exists a  $\mu$ -supra soft regular-open set F containing one of the points x, y but not the other.
- (2) Supra soft  $R_1$ -space if there exist  $\mu$  -supra soft regular -open sets F and G such that  $x \in F$ ,  $y \notin G$  and  $y \in G$ ,  $y \notin F$ .
- (3) Supra soft R-Hausdorff space or supra soft  $-R_2$ -space if there exist  $\mu$  supra soft regular sets F and G such that  $x \in F, y \in G$  and  $F \cap G = \emptyset$ .

**Theorem 3.1:** Every supra soft  $T_i$ -space is supra soft  $-R_i$  for each i = 0,1,2.

**Proof.** Since every supra open soft set is supra R-open in due to [28], then for each i = 0,1,2, supra soft  $T_i$  -space is supra soft  $-R_i$ 

**Remark 3.1:** The converse of Theorem 3.1 is not true in general, as following examples shall show.

**Examples 3.1:** Let  $X = \{h_1, h_2\}$ ,  $E = \{e_1, e_2\}$  and  $\tau = \{\widetilde{\emptyset}, \widetilde{X}, F\}$  where F a soft set is over X defined as follows:  $F(e_1) = \{h_1\}$ ,  $F(e_2) = \{h_2\}$ 

Then $\tau$ , defines a soft topology on X. Consider the associated supra soft topology  $\mu$  with  $\tau$  is defined as  $\mu = \{\widetilde{\emptyset}, \widetilde{X}, G_1, G_2\}$  where  $G_1$  and  $G_2$  are soft sets over X defined as follows:

 $G_1(e_1)=X, G_1(e_2)=\{h_2\},\$ 

 $G_2(e_1) = \{h_1\}, G_2(e_2) = X$ 

Therefore  $(X, \mu, E)$  is supra soft -R<sub>2</sub>space, but it is not supra soft T<sub>2</sub>.

(2) Let  $X = \{h_1, h_2\}$  ,E= $\{e_1, e_2\}$  and  $\tau = \{\widetilde{\emptyset}, \widetilde{X}, F\}$ , where F is a soft set over X defined as follows:  $F(e_1) = \{h_1\}$ ,  $F(e_2) = X$ 

Then  $\tau$  defines a soft topology on X. Consider the associated supra soft topology  $\mu$  with  $\tau$  is defined as $\mu = \{\widetilde{\emptyset}, \widetilde{X}, G_1, G_2\}$ , where  $G_1$  and  $G_2$  are soft sets over X

 $G_1(e_1)=\{h_2\}, G_1(e_2)=\{h_1\}$   $G_2(e_1)=\{h_1\}, G_2(e_2)=X$ 

Therefore  $(X, \mu, E)$  is supra soft  $-R_1$ -space, but it is not supra soft  $T_1$ 

(3) Let  $X = \{h_1, h_2, h_3, h_4, h_5\}$ ,  $E = \{e\}$  and  $\tau = \{\widetilde{\emptyset}, \widetilde{X}, F\}$ , and where F is a soft set over X defined as follows by  $F(e) = \{h_1, h_2, h_3\}$ 

Then  $\tau$ , defines a soft topology on. The associated supra soft topology  $\mu$  with  $\tau$  is defined as  $\mu=\{\widetilde{\emptyset},\widetilde{X},G_1,G_2\}$ , where  $G_1,G_2$  are soft sets over X defined as follows:  $G_1(e_1)=\{h_1,h_2,h_3\},G_1(e_2)=\{h_1,h_2,h_4\}$  Hence  $(X,\mu,E)$  is supra soft  $-R_0$ -space, but it is not supra soft  $-R_0$ . The proof of the next theorem follows immediately from Definition 3.1.

**Theorem 3.2:** Every supra soft -R<sub>i</sub>-space is supra soft -R<sub>i-1</sub> for each i = 1, 2

**Remark 3.2:** The converse of Theorem 3.2 is not true in general, as following examples shall show.

**Examples 3.2:** Let  $X = \{h_1, h_2\}, E = \{e_1, e_2\}, \text{ and } \tau = \{\widetilde{\emptyset}, \widetilde{X}, F\}$  where F is a soft set over X defined as follows:  $F(e_1) = \{h_1\}, F(e_2) = X$ Then  $\tau$  defines a soft topology on X. Consider the associated supra soft topology  $\mu$  with  $\tau$  is defined as  $\mu$ ={ $\widetilde{\emptyset}$ ,  $\widetilde{X}$ ,  $G_1$ ,  $G_2$ ,  $G_3$ }, where  $G_1$ ,  $G_2$  and  $G_3$  are soft sets over X defined as follows:

 $G_1(e_1)=X, G_1(e_2)=\{h_2\},\$ 

 $G_2(e_1) = \{h_1\}, G_2(e_2) = X,$ 

 $G_3(e_1)=\{h_1\},G_3(e_2)=\{h_2\},$ 

Therefore  $(X, \mu, E)$ , is supra soft  $-R_1$  -space, but it is not supra soft  $R_2$ .

**Proposition 3.1:** Let  $(X, \mu, E)$  be a soft topological space and  $x, y \in X$  such that x < y or x > y If there exist  $\mu$  -supra soft regular open soft sets F and G such that  $x \in F$  and  $y \in F^C$  or  $x \in G$  and  $y \in G^C$ .

Then  $(X, \mu, E)$ , is supra soft  $R_0$ -space

**Proof.** Let  $x, y \in X$  such that x < y or x > y. Let F and G be  $\mu$ -supra soft regular open sets such that either  $x \in F$  and  $y \in F^C$  or  $x \in G$  and  $y \in G^C$ . and. If  $x \in F$  and  $y \in F^C$ .

Then  $y \in (F(e))^C$ , for each  $e \in E$ . This implies  $y \in F(e)$  that, for each  $e \in E$ . Therefore  $y \notin F$ . Similarly  $y \in G$ , if and  $x \in G^C$ , then  $x \notin G$ . Hence  $(X, \mu, E)$ , is supra soft  $b - T_0$ -space.

**Proposition 3.2:** Let  $(X, \mu, F)$  be a soft topological space and  $x, y \in X$  such that x < y or x > y If there exist  $\mu$ -supra soft regular open sets F and G such that  $x \in F$  and E are a soft topological space and E and E are a soft topological space and E and E are a soft topological space and E are a soft topological space and E are a soft topological space and E and E are a soft topological space and E and E are a soft topological space and E are a soft topological

Then  $(X, \mu, E)$  is supra soft  $R_1$ -space.

**Proof.** It is similar to the proof of Proposition 3.1.

**Theorem 3.3:** A supra soft topological space  $(X, \mu, E)$  is supra soft  $R_0$ -space if and only if for each pair of distinct points x and y in X,  $cl_{reg}^S(X_E) \neq cl_{reg}^S(Y_E)$ 

**Proof.** Let  $(X, \mu, E)$  be a supra soft  $R_0$  space  $x, y \in X$  such that x < y or x > y. Then, there exists a  $\mu$  -supra soft regular open set F such that  $x \in F$  and  $y \notin F$ . Hence,  $F^C$  is supra soft regular closed set containing y but not x. It follows that  $cl^S_{reg}(Y_E) \subseteq F^C$ ,  $x \notin cl^S_{reg}(Y_E)$  and so.

Thus,  $cl^{S}_{reg}(Y_{E}) \neq cl^{S}_{reg}(X_{E})$ . On the other hand, let x and y be two distinct points in X such that  $cl^{S}_{reg}(Y_{E}) \neq cl^{S}_{reg}(X_{E})$ .

Then, there exists a point Z belongs to one of the sets  $cl^{S}_{reg}$  (Y<sub>E</sub>),  $cl^{S}_{reg}$  (X<sub>E</sub>). but not the other. Say Z  $\in$   $cl^{S}_{reg}$  (Y<sub>E</sub>), and Z  $\notin$ .  $cl^{S}_{reg}$  (X<sub>E</sub>), Now, if x

 $\in {cl^S}_{reg}\left( {{Y_E}} \right)\text{, then,} \\ {cl^S}_{reg}\left( {{Y_E}} \right) \subseteq {cl^S}_{reg}\left( {{X_E}} \right)\text{, which is a contradiction with Z }\notin {\ {cl^S}_{reg}}\left( {{X_E}} \right).$ 

So  $x \notin$  .  $cl_{reg}^{S}(X_{E})$  . Hence  $\left[cl_{reg}^{S}(Y_{E})\right]^{C}$ , is supra soft regular open soft set F containing x but not y. Thus  $(X, \mu, E)$ , is supra soft  $\mu$  – regular – space.

**Theorem 3.4:** A supra soft topological space  $(X, \mu, E)$  is supra soft  $-R_1$ , if  $X_E$  is supra soft regular-closed set in  $\mu$  for each  $x \in X$ .

**Proof.** Suppose that  $x \in X$  and  $X_E$  is supra soft regular-closed set in  $\mu$ . Then  $X_E^C$ , is supra soft regular-closed -open set in  $\mu$ . Let  $x,y \in X$  such that x < y or x > y. For  $x \in X$  and  $X_E^C$  is supra soft regular -open set such that  $x \notin X_E^C$  and  $\in X_E^C$ . Similarly  $y_E^C$  is supra soft regular-open set such that  $y \notin Y_E^C$  and  $x \in y_E^C$ . Thus,  $(X, \mu, E)$  is supra soft -R<sub>1</sub>space.

**Theorem 3.5:** Let  $(X, \mu, E)$  be a supra soft topological space and  $x \in X$ . If  $(X, \mu, E)$  is supra soft regular space, then

- (1)  $x \notin F$  if and only if  $X_E \cap F = \emptyset$  for every  $\mu$  -supra soft regular closed set F
- (2)  $x \notin G$  if and only if  $X_E \cap G = \emptyset$  for every  $\mu$  -supra soft regular closed set G

**Proof.** (1) Let  $(X, \mu, E)$  be a  $\mu$ -supra soft regular-closed soft set such that  $x \notin F$ . Since  $(X, \mu, E)$  is supra soft regular space. By Proposition 3.3 there exists a  $\mu$ -supra soft regular open soft set G such that  $x \in G$  and  $F \cap G = \emptyset$ . It follows that  $X_E \subseteq G$ , from Proposition 3.4 (1)  $X_E \cap F = \emptyset$ . Hence,  $X_E \cap F = \emptyset$ .

Conversely, if  $X_E \cap F = \emptyset$ , then  $x \notin F$  from Proposition 3.4 (2).

(2)Let G be a  $\mu$ -supra soft regular open set such that  $x \notin G$ . If  $x \notin G(e)$  for some  $e \in E$ , then we get the proof.If  $x \notin G(e_1)$  for some  $e_1 \in E$  and  $x \in G(e_2)$  for some  $e_1, e_2 \in E$ . Then  $x \in G^c(e_1)$  and  $x \notin G^c(e_2)$  for some  $e_1, e_2 \in E$ . This means  $X_E \cap G = \emptyset$ . Hence  $G^c$ , is  $\mu$ -supra soft regular closed set such that  $x \notin G^c$ . It follows by (1)  $X_E \cap G^c = \emptyset$ .

This implies that  $X_E \subseteq G$ , and so  $x \in G$ , which is contradiction with  $x \notin G(e_1)$  for some  $e_1 \in E$ . Therefore  $X_E \cap G = \emptyset$ . Conversely  $X_E \cap G = \emptyset$ , if, then it is obvious that  $\notin G$ . This completes the proof.

The next Corollary follows directly from Theorem 3.5.

**Corollary 3.1:** Let  $(X, \mu, E)$  be a supra soft topological space and  $x \in X$ . If  $(X, \mu, E)$  is supra soft regular open space, then the following statements are equivalent:

(1)  $(X, \mu, E)$  is supra soft  $-R_1$  -space.

(2)  $\forall x, y \in X$  such that  $x \neq y$ , there exist  $\mu$  -supra soft regular open soft sets F and G such that  $X_E \subseteq F$  and  $Y_E \cap F = \emptyset$  and  $Y_E \subseteq G$  and  $Y_E \cap G = \emptyset$ .

**Theorem 3.6:** Let  $(X, \mu, E)$  be a supra soft topological space and  $x \in X$ . Then, the following statements are equivalent:

(1)  $(X, \mu, E)$  is supra soft regular open space.

(2) For every  $\mu$  –supra soft regular closed soft set G such that  $X_E \cap G = \emptyset$ , there exist  $\mu$  -supra soft regular open sets  $F_1$  and  $F_2$  such that  $X_E \subseteq F_1$ ,  $G \subseteq F_2$ , and  $F_1 \cap F_2 = \emptyset$ .

<u>Corollary 3.2:</u> The following implications hold from Theorem 3.1, Theorem 3.2 and, Corollary 3.2] for a supra soft topological space (X,  $\mu$ , E) [31].

**Examples 3.3:** Let  $X = \{h_1, h_2\}$ ,  $E = \{e_1, e_2\}$  and  $\tau = \{\widetilde{\emptyset}, \widetilde{X}, F\}$  where F is a soft set over X defined as follows:  $F(e_1) = \{h_1\}$ ,  $F(e_2) = X$ 

Then  $\tau$  defines a soft topology on X. Consider the associated supra soft topology  $\mu$  with  $\tau$  is defined as  $\mu$ ={ $\widetilde{\emptyset}$ ,  $\widetilde{X}$ ,  $G_1$ ,  $G_2$ ,  $G_3$ }, where  $G_1$ ,  $G_2$  and  $G_3$  are soft sets over X defined as follows:

 $G_1(e_1)=X, G_1(e_2)=\{h_2\},$ 

 $G_2(e_1)=\{h_1\},G_2(e_2)=X,$ 

 $G_3(e_1)=\{h_1\},G_3(e_2)=\{h_2\},$ 

Therefore, is supra soft  $R_1\text{-space, On the other hand, we note that for the singleton soft points <math display="inline">h_1$  and  $h_2,$  where

 $h_1(e_1)=\{h_1\}, h_1(e_2)=\{h_1\},$ 

 $h_2(e_1) = \{h_2\}, h_2(e_2) = \{h_2\},$ 

The relative complement  $h_1^C$  and  $h_2^C$ , where

 $h_1^C(e_1) = \{ h_2 \}, h_1^C(e_2) = \{ h_2 \},$ 

 $h_1^C(e_1) = \{h_1\}, h_1^C(e_2) = \{h_1\},$ 

Thush  $_{ZE}^C$  , is not  $\mu$  -supra soft regular open soft set. This shows that, the converse of the above theorem does not hold.

Also, we have  $\mu_{1e}$ ={X,Ø,{ $h_1$ }} and  $\mu_{2e}$ ={X,Ø,{ $h_1$ },{ $h_2$ }} Therefore (X,  $\mu_{1e}$ ), is not a supra R<sub>1</sub>-space, at the time that (X,  $\mu$ , E) is a supra soft-R<sub>1</sub>space.

**Definition 3.2:** Let  $(X, \tau, E)$  be a soft topological space and  $\mu$  be an associated supra soft topology with  $\tau$ . Let G be a  $\mu$ -supra soft regular closed set in X and  $x \in X$  such that  $x \notin G$ . If there exist  $\mu$  -supra soft regular open soft sets  $F_1$  and  $F_2$  such that  $F_1$  and  $F_2$  and  $F_3$  such that  $F_4$  and  $F_5$  such that  $F_6$  and  $F_7$  supra soft  $F_8$  and  $F_9$  is called supra soft  $F_8$  regular space. A supra soft  $F_8$  regular  $F_8$  space is called supra soft  $F_8$  space.

The proofs of the next propositions are obvious.

**Proposition 3.3:** Let  $(X, \tau, E)$  be a soft topological space and  $\mu$  be an associated supra soft topology with  $\tau$ . Let G be a  $\mu$  -supra soft regular closed soft set in X and  $x \in X$  such that  $x \notin G$ . If  $(X, \mu, E)$  is supra soft R-regular space, then there exists a  $\mu$ -supra Soft regular-open set F such that  $x \in F$  and  $F \cap G = \emptyset$ 

**Proposition 3.4:** Let  $(X, \mu, E)$  be a supra soft topological space  $F \in S_E(X)$ , and  $x \in X$ . Then,

- (1)  $x \in F$  if and only if  $X_E \subseteq F$ .
- (2) If  $X_E \cap F = \emptyset$ , then  $x \notin F$ .

Proof.

(1)  $\Rightarrow$  (2) Let G be a  $\mu$  -supra soft regular closed soft set such that  $X_E \cap G = \emptyset$ .

Then  $x \notin G$ , from Theorem 3.5 (1). It follows by (1), there exist  $\mu$  -supra soft regular open soft sets  $F_1$  and  $F_2$  such that  $\in F_1$ ,  $G \subseteq F_2$  and  $F_1 \cap F_2 = \emptyset$ .

This means that  $X_E \subseteq F_1$ ,  $G \subseteq F_2$  and  $F_1 \cap F_2 = \emptyset$ .

(2) ⇒ (1) Let be a μ-supra soft regular closed soft set such that ∉ G. Then X<sub>E</sub>∩G =Ø, from Theorem 3.5 (1). It follows by (2), there exist μ-supra soft regular open soft setsF<sub>1</sub> and F<sub>2</sub> such that X<sub>E</sub>⊆F<sub>1</sub>, G⊆F<sub>2</sub>, F<sub>1</sub> ∩ F<sub>2</sub> = Ø. Hence, x ∈ F<sub>1</sub>, and G ⊆ F<sub>2</sub>. F<sub>1</sub> ∩ F<sub>2</sub> = Ø. Thus (X, μ, E), is supra soft R-regular space.

**Theorem 3.7:** Let  $(X, \mu, E)$  be a supra soft topological space. If  $(X, \mu, E)$  is supra soft  $R_3$ -space, then  $\forall x \in X_E$ , is  $\mu$ -supra soft regular closed set.

**Proof.** We are going to prove that is  $x_E^C$ -supra  $\mu$ -soft regular open set  $y \in \{x\}^C$  for each.

Since  $(X, \mu, E)$  is supra soft  $-R_3$ -space. Then, by Theorem 3.6, there exist  $\mu$  –supra soft regular open soft sets  $F_y$  and G such That  $y_E \subseteq F_y$  and  $x_E \cap F_y = \emptyset$  and  $x_E \subseteq G$  and  $y_E \cap G = \emptyset$ .

It follows that  $\cup_{y\in\{x\}^C}\ F_y\subseteq x_E^C$  , Now,  $x_E^C\subseteq \cup_{y\in\{x\}^C}\ F_y$  we want to prove that

Let  $\cup_{y \in \{x\}^C} F_y$ =H, where H(e)=  $\cup_{y \in \{x\}^C} F(e)_y$  for each  $e \in E$ .

Since  $x_E^C(e) = \{x\}^C$  for each  $e \in E$  from Definition 2.4. So for each  $y \in \{x\}^C$  and  $e \in E$ ,  $x_E^C(e) = \{x\}^C = \cup_{y \in \{x\}^C} \{y\} = \cup_{y \in \{x\}^C} y_E(e) \subseteq \cup_{y \in \{x\}^C} F(e)_y = H(e)$  Thus,  $x_E^C \subseteq \cup_{y \in \{x\}^C} F_y$ ,  $x_E^C = \cup_{y \in \{x\}^C} F_y$ This means that  $x_E^C$ , is  $\mu$ -supra soft regular-open set for each  $y \in \{x\}^C$ . Therefore,  $X_E$ , is  $\mu$ -supra soft regular closed set

**Theorem 3.8:** Every supra soft regular  $R_3$ ,-space is supra soft regular  $R_2$ space.

**Proof.** Let  $(X, \mu, E)$  be a supra soft regular  $R_3$ -space and  $x, y \in X$  such that x < y or x > y

By Theorem 3.7,  $Y_E$  is  $\mu$  -supra soft regular closed soft set and  $x \notin Y_E$ . It follows from the supra soft regular-regularity, there exist  $\mu$  supra soft regular-open soft sets  $F_1$  and  $F_2$  such that  $x \in F_1$ ,  $y \in F_2$  and  $F_1 \cap F_2 = \emptyset$ . Thus,  $x \in F_1$ ,  $y \in y_E \subseteq F_2$  and  $F_1 \cap F_2 = \emptyset$ .

Therefore  $(X, \mu, E)$ , is supra soft regular  $R_2$ -space.

**Definition3.3:** Let  $(X, \mu, E)$  be a supra soft topological space, F and G be  $\mu$ -supra soft regular closed soft sets in X such that  $F \cap G = \emptyset$  If there exist  $\mu$ -supra soft regular-open sets  $F_1$  and  $F_2$  such that  $F \subseteq F_1$ ,  $G \subseteq F_2$ , and  $F_1 \cap F_1 = \emptyset$ , then  $(X, \mu, E)$  is called supra soft regular-normal space.

A supra soft regular-normal regular- $\mathbf{R_1}$ -space is called a supra soft regular  $\mathbf{R_4}$ -space.

**Theorem 3.9:** Let  $(X, \mu, E)$  be a supra soft topological space and  $x \in X$ . Then, the following statements are equivalent:

(1)  $(X, \mu, E)$  is supra soft -normal space.

(2) For every  $\mu$  -supra soft regular-closed soft set F and G  $\mu$  -supra soft regular open soft set such that  $F \subseteq G$ , there exists a  $\mu$  -supra soft regular open soft set  $F_1$  such that  $F \subseteq F_1$ ,  $\operatorname{cl}^S_{\operatorname{reg}}(F) \subseteq G$ .

### Proof.

(1) $\Rightarrow$ (2) Let F be a  $\mu$ -supra soft regular closed set and G be a  $\mu$ -supra soft regular open set such that  $F\subseteq G$  Then  $F,G^C$  are  $\mu$ -supra soft regular closed soft sets such that  $F\cap G^C=\emptyset$ .

It follows by (1), there exist  $\mu$ -supra soft regular open soft sets  $F_1$  and  $F_2$  such that  $F \subseteq F_1$ ,  $G^C \subseteq F_2$  and  $F_1 \cap F_2 = \emptyset$  Now, $F_1 \subseteq F_2^C$ , so

 ${cl^S}_{reg}$   $({F_1})\subseteq {cl^S}_{reg}$   $({F_2}^C)$  where G is  $\mu$  –supra soft regular-open soft set. Also,  ${F_2}^C\subseteq G$ .

Hence,  $\operatorname{cl}^{S}_{\operatorname{reg}}(\mathbf{F_{1}}) \subseteq \mathbf{F_{2}}^{C} \subseteq G$ . Thus  $F \subseteq \mathbf{F_{1}}$ ,  $\operatorname{cl}^{S}_{\operatorname{reg}}(\mathbf{F_{1}}) \subseteq G$ 

 $(2)\Rightarrow (1)$  Let  $G_1$ ,  $G_2$  be  $\mu$ -supra soft regular closed soft sets such that  $G_1\cap G_2=\emptyset$  Then,  $G_1\subseteq (G_2)^C$ , then by hypothesis, there exists a  $\mu$ -supra soft regular open soft set  $F_1$  such that  $G_1\subseteq F_1$ ,  $\operatorname{cl}^S_{\operatorname{reg}}(F_1)\subseteq (G_2)^C$  So,  $G_2\subseteq [\operatorname{cl}^S_{\operatorname{reg}}(F_1)]^C$ ,  $G_1\subseteq F_1$  and  $[\operatorname{cl}^S_{\operatorname{reg}}(F_1)]^C\cap F_1=\emptyset$ , where  $F_1$  and  $[\operatorname{cl}^S_{\operatorname{reg}}(F_1)]^C$  are  $\mu$ -supra soft regular open soft sets. Thus,  $(X,\mu,E)$  is supra soft normal space.

### 4. SUPRA R-IRRESOLUTE SOFT FUNCTIONS

**<u>Definition4.1:</u>** Let  $(X, \tau_1, E)$  and  $(Y, \tau_2, K)$  be soft topological spaces,  $\mu_1$  and  $\mu_2$  be associated supra soft topologies with  $\tau_1$  and  $\tau_2$ , respectively. The soft function  $f_{P\mu}: S_E(X) \to S_E(Y)$  is called

- (1) Supra soft regular open soft if  $f_{P\mu}(F){\in}\mbox{ SROS}_E(\ \mu_1\ )\ \mbox{ for each } F{\in}\ \tau_1$
- (2) Supra soft regular irresolute soft if f  $^{-1}{}_{P\mu}$  (F)  $\in$  SROS  $_E($   $\mu_1$  ) for each F  $\in$  SROS  $_K($   $\mu_2)$
- (3) Supra soft regular irresolute open soft if  $f_{P\mu}(F) \in SROS_K(\mu_2)$  for each  $F \in SROS_E(\mu_1)$

**Theorem 4.1:** Let  $(X, \tau_1, E)$  and  $(Y, \tau_2, K)$  be soft topological spaces,  $\mu_1$  and  $\mu_2$  be associated supra soft topologies with  $\tau_1$  and  $\tau_2$ , respectively and  $f_{\mu P}: S_E(X) \to S_E(Y)$  be a soft function which is Bijective and supra-irresolute regular open soft. If  $(X, \mu_1, E)$  is supra soft  $R_0$ space, then  $(Y, \mu_2, K)$  is also a supra soft  $R_0$ space.

**Proof.** Let  $\mathbf{y_1},\mathbf{y_2}\in Y$  such that  $\mathbf{y_1}\neq \mathbf{y_2}.$  Since  $\mathbf{f_{\mu P}}$  is surjective, then there exist  $\mathbf{x_1},\mathbf{x_2}\in X$  such that  $\mu(\mathbf{x_1})=\mathbf{y_1},\mu(\mathbf{x_2})=\mathbf{y_2}$  and  $\mathbf{x_1}\neq \mathbf{x_2}.$  By hypothesis, there exist  $\mu_1$ -supra soft regular open soft sets F and G in X such that either  $\mathbf{x_1}\in X$  and  $\mathbf{x_1}\notin F$ , or  $\mathbf{x_2}\in G$  and  $\mathbf{x_1}\notin G$ .

So, either  $\mathbf{x}_1 \in \mathcal{K}_{\mathrm{att}}$  and  $\mathbf{x}_1 \notin \mathcal{F}_{\mathrm{e}}(\mathbf{e})$  and  $\mathbf{x}_2 \notin \mathcal{F}_{\mathrm{E}}(\mathbf{e})$  or  $\mathbf{x}_2 \in \mathcal{G}_{\mathrm{E}}(\mathbf{e})$  and  $\mathbf{x}_1 \notin \mathcal{G}_{\mathrm{E}}(\mathbf{e})$  for each  $\mathbf{e} \in \mathbb{F}$ .

This implies that, either  $y_1 = \mu(x_1) \in \mu[F_E(e)]$  and  $y_2 = \mu(x_2) \notin \mu[F_E(e)]$  or

 $\mathbf{y}_2 = \mu(\mathbf{x}_2) \in \mu[\mathbf{G}_E(\mathbf{e})]$  and  $\mathbf{y}_1 = \mu(\mathbf{x}_1) \notin \mu[\mathbf{G}_E(\mathbf{e})]$  for  $\mathbf{e} \in E$  Hence, either  $\mathbf{y}_1 \in f_{\mu P}(F)$  and  $\mathbf{y}_2 \notin f_{\mu P}(F)$  or  $\mathbf{y}_2 \in f_{\mu P}(G)$  and  $\mathbf{y}_1 \notin f_{\mu P}(F)$ . Since  $f_{\mu P}$  is supra soft irresolute regular open function, then  $f_{\mu P}(F)$ ,  $f_{\mu P}(G)$  are supra soft regular open sets in Y. Hence,  $(Y, \mu_2, K)$  is also a supra soft-  $\mathbf{R}_0$  space **Theorem 4.2:** Let  $(X, \tau_1, E)$  and  $(Y, \tau_2, K)$  be soft topological spaces,  $\mu_1$  and  $\mu_2$  be associated supra soft topologies with  $\tau_1$  and  $\tau_2$ , respectively. Let  $f_{\mu P}: S_E(X) \to S_E(Y)$  be a soft function which is bijective and supra soft regular irresolute open soft. If  $(X, \mu_1, E)$  is supra soft  $\mathbf{R}_1$ -space, then  $(Y, \mu_2, K)$  is also a supra soft- $\mathbf{R}_1$ space.

**Proof.** It is similar to the proof of Theorem 4.1.

**Theorem 4.3:** Let  $(X, \boldsymbol{\tau}_1, E)$  and  $(Y, \boldsymbol{\tau}_2, K)$  be soft topological spaces,  $\mu_1$  and  $\mu_2$  be associated supra soft topologies with  $\boldsymbol{\tau}_1$  and  $\boldsymbol{\tau}_2$ , respectively. Let  $f_{\mu P}: S_E(X) \to S_E(Y)$  be a soft function which is Bijective and supra soft regular -irresolute open soft. If  $(X, \mu_1, E)$  is supra soft- $\boldsymbol{R}_2$ -space, then  $(Y, \mu_2, K)$  is also a supra soft  $\boldsymbol{R}_2$ -space.

**Proof.** Let  $y_1,y_2\in Y$  such that  $y_1\neq y_2$ . Since  $f_{Mp}$  is surjective, then there exist  $x_1,x_2\in X$  such that  $\mu(x_1)=y_1,\ \mu(x_2)=y_2$  and  $x_1\neq x_2$ . and. By hypothesis, there exist  $\mu_1$ -supra soft regular -open soft sets F and G in X such that  $x_1\in F$ ,  $x_2\in G$  and  $F\cap G=\emptyset$ . So  $x_1\in F_E(e)$ ,  $x_2\in G_E(e)$ , and  $G_E(e)\cap F_E(e)=\emptyset$  for each  $e\in E$ . This implies that  $y_1=\mu(x_1)\in \mu[F_E(e)]$  and  $y_2=\mu(x_2)\in \mu[G_E(e)]$  for each  $e\in E$ . Hence,  $y_1\in f_{\mu P}(F)$ ,  $y_2\in f_{\mu P}(G)$   $f_{\mu P}(F)\cap f_{\mu P}(G)=f_{\mu P}(F\cap G)=f_{\mu P}[\emptyset_E]=\emptyset_K$ .

Since  $f_{\mu P}$  is supra soft regular irresolute open soft function, then  $f_{\mu P}(F)$ ,  $f_{\mu P}(G)$  are supra soft regular -open sets in Y. Thus  $(Y, \mu_2, K)$  is also a supra soft- $\mathbf{R}_2$ -space.

**Theorem 4.4:** Let  $(X, \tau_1, E)$  and  $(Y, \tau_2, K)$  be soft topological spaces  $\mu_1$  and  $\mu_2$  be associated supra soft topologies with  $\tau_1$  and  $\tau_2$ , respectively. Let  $f_{P\mu}: S_E(X) \to S_E(Y)$  be a soft function which is bijective, supra soft regular - irresolute soft and supra soft regular -irresolute open soft.  $(X, \mu_1, E)$  is supra soft Regular - regular -space, then  $(Y, \mu_2, K)$  is also a supra soft Regular - regular space.

**Proof.** Let G be a supra b-closed soft set in Y and  $y \in Y$  such that  $y \notin G$ . Since  $f_{\mathbb{P}\mu}$  is surjective and supra soft regular -irresolute soft, then there exist  $x \in X$  such that  $\mu(x) = y$  and  $f^{-1}_{\mathbb{P}\mu}(G)$  is supra soft regular -closed soft set in X such that  $x \notin f^{-1}_{\mathbb{P}\mu}(G)$ . By hypothesis, there exist  $\mu_1$ -supra soft regular-open soft sets F and H in X such that  $x \in F$ ,  $f^{-1}_{\mathbb{P}\mu}(G) \subseteq H$  and  $F \cap H = \emptyset$ . It follows that  $\mathbf{x} \in \mathbf{F}_{\mathbf{E}}(\mathbf{e})$ , for each  $\mathbf{e} \in E$  and  $G = f_{\mathbb{P}\mu}[f^{-1}_{\mathbb{P}\mu}(G)] \subseteq f_{\mathbb{P}\mu}(H)$  So  $\mathbf{y} = \mu(x) \in \mu[\mathbf{F}_{\mathbf{E}}(\mathbf{e})]$  for each  $\mathbf{e} \in E$  and  $\mathbf{f} \in F_{\mathbb{P}\mu}(H)$ .

Hence  $f_{P\mu}(F) \cap f_{P\mu}(H) = f_{P\mu}[F \cap H] = f_{P\mu}[\emptyset_E] = \emptyset_K$ ,.

Since  $f_{P\mu}$  is supra soft regular -irresolute open soft function. Then  $f_{P\mu}(F)$ ,  $f_{P\mu}(H)$  are supra soft regular -open soft sets in Y. Thus  $(Y, \mu_2, K)$ , is also a supra soft Regular -regular space.

The next Corollary follows immediately from Theorem 4.2 and Theorem 4.4.

**Corollary 4.1:** Let(X,  $au_1$ , E) and  $(Y, au_2, K)$  be soft topological spaces  $\mu_1$  and  $\mu_2$  be associated supra soft topologies with  $au_1$  and  $au_2$ , respectively. Let  $f_{P\mu}: S_E(X) \to S_E(Y)$  be a soft function which is bijective, supra soft regular -irresolute soft and supra soft regular -irresolute open soft. If  $(X, \mu_1, E)$  is supra soft  $-\mathbf{R}_3$ -space, then  $(Y, \mu_2, K)$  is also a supra soft  $-\mathbf{R}_3$ -space.

**Theorem 4.5:** Let  $(X, \tau_1, E)$  and  $(Y, \tau_2, K)$  be soft topological spaces  $\mu_1$  and  $\mu_2$  be associated supra soft topologies with  $\tau_1$  and  $\tau_2$ , respectively. Let  $f_{\mathbb{P}\mu}: S_E(X) \to S_E(Y)$  be a soft function which is bijective, supra soft regular-irresolute soft and supra soft regular-irresolute open soft. If  $(X, \mu_1, E)$  is supra soft regular - normal -space, then  $(Y, \mu_2, K)$  is also a supra soft regular- normal space.

**Proof.** Let F,G be supra soft regular -closed soft sets in Y such that  $F \cap G = \emptyset_K$ 

Since  $f_{P\mu}$  is supra soft regular-irresolute soft, then  $f^{-1}_{P\mu}(F), f^{-1}_{P\mu}(G)$  and are supra soft regular-closed soft set in X such that  $f^{-1}_{P\mu}(F) \cap f^{-1}_{P\mu}(G) = f^{-1}_{P\mu}[F \cap G] = f^{-1}_{P\mu}[\phi_K] = \phi_E$ 

By hypothesis, there exist supra soft regular-open soft sets M and H in X such that  $f^{-1}{}_{P\mu}(F)\subseteq M$ ,  $f^{-1}{}_{P\mu}(G)\subseteq H$  and  $F\widetilde{\cap}\ G=\emptyset_E$  It follows that,  $F=f_{P\mu}[\ f^{-1}{}_{P\mu}(F)]\subseteq f_{P\mu}(M)$ ,  $G=f_{P\mu}[\ f^{-1}{}_{P\mu}(G)]\subseteq f_{P\mu}(H)$  and

 $f_{P\mu}$  (M) $\widetilde{\cap}$   $f_{P\mu}$  (H)=  $f_{P\mu}$  [M  $\widetilde{\cap}$  H]=  $f_{P\mu}$  [ $\emptyset_E$ ] =  $\emptyset_K$ . Since  $f_{P\mu}$  is supra soft regular -irresolute open soft function. Then  $f_{P\mu}$ (M),  $f_{P\mu}$ (H) are supra soft regular-open soft sets in Y. Thus (Y,  $\mu_2$ , K), is also a supra soft regular-

normal space.

Corollary 4.2: Let  $(X, \tau_1, E)$  and  $(Y, \tau_2, K)$  be soft topological spaces,  $\mu_1$  and  $\mu_2$  be associated supra soft topologies with  $\tau_1$  and  $\tau_2$ , respectively. Let  $f_{P\mu}: S_E(X) \to S_E(Y)$  be a soft function which is bijective, supra soft regular-irresolute soft and supra soft regular-irresolute open soft. If  $(X, \mu_1, E)$  is supra soft- $R_4$ -space, then  $(Y, \tau_2, K)$  is also a supra soft- $R_4$ -space.

**Proof.** It is obvious from Theorem 4.2 and Theorem 4.5.

### 5. CONCLUSION

In this paper, we introduced and investigated some soft separation axioms by using the notion of supra –soft regular open soft sets, which is a generalization of the supra soft separation axioms mentioned. We studied the relationships between these new soft separation axioms. We showed that, some classical results in supra soft topological space are not true. We hope that, the results in this paper will help researcher enhance and promote the further study on supra soft topology to carry out a general framework for their applications in practical life.

### CONFLICT OF INTEREST

No conflict of interest was declared by the authors.

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